

6.7 Videos Guide

6.7a

Definitions: (hyperbolic functions)

$$\begin{aligned} \circ \sinh x &= \frac{e^x - e^{-x}}{2} & \text{csch } x &= \frac{1}{\sinh x} \\ \circ \cosh x &= \frac{e^x + e^{-x}}{2} & \text{sech } x &= \frac{1}{\cosh x} \\ \circ \tanh x &= \frac{\sinh x}{\cosh x} & \text{coth } x &= \frac{\cosh x}{\sinh x} \end{aligned}$$

6.7b

Exercises:

- Find the numerical value of each expression.
a) $\sinh 4$ b) $\sinh(\ln 4)$
- Prove the identity.
 $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

6.7c

- Derivatives of hyperbolic functions
 - $\frac{d}{dx}(\sinh x) = \cosh x$
 - $\frac{d}{dx}(\cosh x) = \sinh x$
 - $\frac{d}{dx}(\tanh x) = \text{sech}^2 x$
 - $\frac{d}{dx}(\text{sech } x) = -\text{sech } x \tanh x$
 - $\frac{d}{dx}(\text{csch } x) = -\text{csch } x \coth x$
 - $\frac{d}{dx}(\coth x) = -\text{csch}^2 x$
 - Note that reversing these gives integration rules

Exercises:

- Find the derivative. Simplify where possible.
 $F(t) = \ln(\sinh t)$
- Evaluate the integral.

$$\int \frac{\text{sech}^2 x}{2 + \tanh x} dx$$

6.7d

- Inverse hyperbolic functions
 - $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}), \quad x \in \mathbb{R}$
 - $\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}), \quad x \geq 1$
 - $\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right), \quad -1 < x < 1$

- Derivatives of inverse hyperbolic functions

- $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$
- $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$
- $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}, \quad |x| < 1$

$$\frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$$
$$\frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$$
$$\frac{d}{dx}(\operatorname{coth}^{-1} x) = \frac{1}{1-x^2}, \quad |x| > 1$$

Exercise:

- Evaluate the integral.

$$\int_0^1 \frac{1}{\sqrt{16t^2 + 1}} dt$$